Grade 1 Mathematics, Quarter 4, Unit 4.1 Adding Multiple Numbers to Solve Word Problems

Number of instructional days:

10 (1 day = 45-60 minutes)

Content to be learned

- Add three whole numbers whose sum is less than 20.
- Use a variety of representations to act out the problem.

Mathematical practices to be integrated

Reason abstractly and quantitatively.

• Design a coherent representation of the problem.

Model with mathematics.

- Apply mathematical practices to everyday situations.
- Recognize the need to make revisions in the design if needed.

- How can you use objects, drawings, or equations to solve this problem?
- What is the unknown addend or subtrahend in this equation?

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Common Core State Standards for Mathematical Content

Operations and Algebraic Thinking

1.0A

Represent and solve problems involving addition and subtraction.

1.OA.2 Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.

Work with addition and subtraction equations.

1.OA.7 Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. For example, which of the following equations are true and which are false? 6 = 6, 7 = 8 - 1, 5 + 2 = 2 + 5, 4 + 1 = 5 + 2.

Common Core Standards for Mathematical Practice

2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents— and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

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Clarifying the Standards

Prior Learning

Kindergarten students used concrete objects and acted out situations to solve addition and subtraction problems. These problems were real-world situations. Students learned the mathematical language to support their discussion of their representations.

Current Learning

First-grade students use their knowledge of addition and subtraction to compare, represent, and solve for unknowns, and extend addition and subtraction to solve problems with three whole numbers whose sum is less than or equal to 20. They learn to recognize the properties of addition and subtraction and how to use them to solve problems in an easier way.

Future Learning

In second grade, students will add up to four two-digit numbers using strategies involving place value and operations. They will add and subtract within 1,000 using concrete models or drawing and strategies.

Additional Findings

According to the *Progressions for Common Core Mathematics*, "although the progressions in OA are organized around whole numbers, it is more general because it describes concepts, properties, and representations that extend to other number systems, to measures, and to algebra."

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Grade 1 Mathematics, Quarter 4, Unit 4.2 Using Place Value to Add Combinations of 1- and 2-Digit Addends

Overview

Number of instructional days:

10 (1 day = 45-60 minutes)

Content to be learned

- Understand place value with adding 2 digit numbers and a one digit number and 2 digit numbers and a multiple of 10.
- Apply the properties of addition with adding a 2 digit number and a one digit number and adding a 2 digit number and a multiple of 10 within 100.
- Understand the relationship between addition and subtraction.
- Understand different strategies to add within 100.
- Understand the meaning of the equal sign.

Mathematical practices to be integrated

Model with mathematics.

- Students identify quantities in practical situations
- Students solve problems that arrive with everyday life
- Students state the meanings of the symbols they choose including using the equal sign consistently and appropriately.

Look for and express regularity in repeated reasoning.

- Students notice repeated calculations.
- Students look for and use the fact families, multiples of 10s as short cuts.

- How are addition and subtraction similar/different?
- How do you use the values of tens and ones to add numbers?
- How and which concrete models and drawings do you use to add and subtract numbers?
- How does counting by tens help you add numbers?

- What strategy was used to add ? To subtract ?
- What strategy did you use to add ? To subtract ?
- How do you know if an equation is true or false?

Common Core State Standards for Mathematical Content

Number and Operations in Base Ten

Use place value understanding and properties of operations to add and subtract.

1.NBT.4 Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten.

Operations and Algebraic Thinking

1.0A

1.NBT

Work with addition and subtraction equations.

1.OA.7 Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. For example, which of the following equations are true and which are false? 6 = 6, 7 = 8 - 1, 5 + 2 = 2 + 5, 4 + 1 = 5 + 2.

Common Core Standards for Mathematical Practice

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

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8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation (y - 2)/(x - 1) = 3. Noticing the regularity in the way terms cancel when expanding (x - 1)(x + 1), $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Clarifying the Standards

Prior Learning

In kindergarten, teachers helped children lay the foundation for understanding the base-ten system by drawing special attention to 10. They decomposed 10 into pairs such as 9+1, 2+8, 3+7 and found the number that makes 10 when added to a given number.

Current Learning

Children in first grade use mathematical reasoning, such as the associative property. Students learn to view ten ones as a unit called a ten. The ability to compose and decompose this unit flexibly and to view the numbers 11–19 as composed of one ten and some ones allows development of efficient, general baseten methods for addition and subtraction. Students see a two-digit numeral as representing some tens and they add and subtract using this understanding.

First graders use their base-ten work to compute sums within 100 with understanding. Concrete objects, cards, or drawings afford connections with written numerical work and discussions and explanations in terms of tens and ones. Showing composition of a ten with objects or drawings connects the visual ten with the written numeral 1 that indicates 1 ten.

Adding tens and ones separately is a general method that can extend to any sum of multi-digit numbers. Students use counting on strategies to count on by tens and ones or they add instead of counting the tens and ones.

Future Learning

In grade 2 students will extend their base-ten understanding to hundreds. They will add and subtract within 1000 with composition and decomposition and be able to explain the reasoning and processes they use. They become fluent with addition and subtraction within 100. In grade 3 students become fluent with addition and subtraction within 1000. They become fluent with strategies and algorithms that are based on place value, properties of operations, an/or the relationship between addition and subtraction. In grade 4 students use standard algorithms to fluently add and subtract. They also use methods based on place value and properties to multiply and divide with multi-digit numbers.

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Additional Findings

According to *the Research Companion to Principals and Standards*, "research on instructional programs in United States, Europe, and South Africa indicates that focusing on understanding multi-digit addition and subtraction methods results in much higher levels of correct multi-digit methods and produces children who can explain how they got their answers using quality language." (p. 79)

Furthermore, *The Research Companion to Principals and Standards states that* students can use roughly three classes of effective methods for multi-digit addition and subtraction, although some methods are mixtures. *Counting list methods* are extensions of single-digit counting methods. Children initially may count large numbers by ones, but these unitary methods are highly inaccurate and are not effective. All children need to be helped as rapidly as possible to develop prerequisites for methods using tens. In counting list methods using tens, children count on or count up by tens and by ones. These methods generalize readily to counting on or up by hundreds, but become unwieldy for lager numbers. In *decomposing methods*, children decompose numbers so that they can add or subtract like units (add tens to tens, one to ones, hundreds to hundreds, etc.). These methods generalize easily to very large numbers. *Recomposing methods* are like a make-a ten or doubles methods. The solver changes both numbers by giving some amount of one number to another number (i.e., in adding) or by changing both numbers equivalently to maintain the same difference (i.e. in subtracting). (p. 79)

Additionally, students must understand two aspects of multi-digit numbers (1) they must add like units together, and (2) when they get 10 or more of anything they must give 1 group of ten of those things to the next left place and record the remaining things. The second understanding has been called "carrying" or "regrouping" or "trading" *Research Companion to Principals and Standards for Schools Mathematics p.80*.

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Grade 1 Mathematics, Quarter 4, Unit 4.3 Comparing 2-Digit Numbers

Overview

Number of instructional days:

10 (1 day = 45–60 minutes)

Content to be learned

- Compare the relationship of two 2-digit numbers.
- Understand the meaning of symbols >, <, and =.
- Accurately use these symbols in comparing two 2-digit numbers.

Mathematical practices to be integrated

Model with mathematics.

- Write an equation to describe how two 2-digit numbers as they apply to everyday situations.
- Use models to verify their equation.
- Re-state the meaning of the symbols they choose to use in the equation.

Attend to precision.

- Accurately explain your reasoning.
- Communicate clearly to others.

- What is the relationship of these two sets of objects?
- How can grouping these objects in bundles of 10 help in comparing the two sets?
- How can we compare these two numbers using a model?
- How would you write your explanation in the form of an equation?
- What is the meaning of the following symbols: <, >, and = ?

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Common Core State Standards for Mathematical Content

Number and Operations in Base Ten

1.NBT

Understand place value.

1.NBT.3 Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >, =, and <.

Common Core Standards for Mathematical Practice

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Clarifying the Standards

Prior Learning

Kindergarteners are able to group sets of 11-19 objects into ten and some ones.

They learn to view the numbers 11-19 as tens and ones. They use objects, drawings, and equations to describe, explore, and explain how the "teen" numbers are composed of a "ten" and some "ones."

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Current Learning

In grade 1, students rename 10 ones as a "ten." First graders learn the two digits of a two-digit number represent amounts of tens and ones. Students use their experiences with "tens" to recognize that the digit in the "tens " place is more important for determining the worth of a two-digit number. They use this understanding to compare two-digit numbers and note the difference with the symbols >, <, and =.

Future Learning

In future grades, students extend their knowledge of base ten is to hundreds. They use this knowledge to add, subtract, multiply, and divide. The methods they use to apply operations using multiple digit numbers are supported by their understanding of place value.

Additional Findings

According to *Principles and Standards for School Mathematics*, it is essential that students develop a solid understanding of the base-ten numeration system and place value concepts by the end of Grade 2. Students need many instructional experiences to develop their understanding of the system, including how numbers are written.

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Grade 1 Mathematics, Quarter 4, Unit 4.4 Connecting Numbers to Linear Measurement

Overview

5

Number of instructional days:

(1 day = 45-60 minutes)

Content to be learned

- Express the length of an object in terms of iterating units
- Measure the length of an object with accuracy; leaving no gaps or overlays
- Use whole numbers to express the length of an object
- Understand that the objects used to measure need to be smaller in size than the object being measured.

Essential questions

- What can you use to measure this object?
- What are the rules we need to measure accurately?

Mathematical practices to be integrated

Model with mathematics.

- Identify appropriate unit(s) to measure an object.
- Explain how the choice of unit is reasonable.
- Recognize when the unit is not reasonable.
- Apply measuring to everyday situations.

Use appropriate tools strategically.

- Identify and use appropriate measuring tools.
- How would you use numbers to talk about the length of the object?
- What other units can use to measure the same object?

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Common Core State Standards for Mathematical Content

Measurement and Data

1.MD

Measure lengths indirectly and by iterating length units.

- 1.MD.1. Order three objects by length; compare the lengths of two objects indirectly by using a third object.
- 1.MD.2. Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. *Limit to contexts where the object being measured is spanned by a whole number of length units with no gaps or overlaps.*

Common Core Standards for Mathematical Practice

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

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Clarifying the Standards

Prior Learning

In kindergarten, students compare and order objects by their measurable attributes. They compare the length of objects directly to each other and indirectly through comparing both objects to a third object.

Current Learning

First grade students continue to build a background for measurement by exploring the properties of congruence and symmetry. They measure by laying multiple copies of a unit end to end and count the number of units used.

Routine: measurement should be integrated into other areas such as other math units, science and social studies.

Future Learning

Second graders use rulers and other measuring tools to support their understanding the iteration of units. They collect measurement to use for data. In third grade, students begin to work with fraction parts of units. By the end of elementary school, students extend their knowledge of measurement finding the area of a two dimensional objects.

Additional Findings

According to *Principles and Standards for School Mathematics* (p. 103), measurement is one of the most widely used applications of mathematics. It bridges geometry and numbers. A foundation in measurement concepts that enables students to use measurement systems, tools, and techniques should be established through direct experiences with comparing objects, counting units, and making connections between spatial concepts and numbers.

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Grade 1 Mathematics, Quarter 4, Unit 4.5 Connecting the Concepts of Equal Parts/Shares to Fractional Terms

Overview

5

Number of instructional days:

(1 day = 45-60 minutes)

Content to be learned

- Divide circles and squares into two and four equal pieces.
- Express shares and parts of a whole with the terms fourths, half, and quarters.
- Use the phrase half of, quarter of, and a fourth of to describe parts of a whole.
- Understand that the more an object is divided, the smaller the parts will be.

Mathematical practices to be integrated

Make sense of problems and persevere in solving them.

- Use an object (create a model) to show your reasoning.
- Use models to show how different fractional parts are related to each other.

Use appropriate tools strategically.

- Draw a picture of a model.
- Use measuring tools to check for accuracy.

- How can we share a whole (object) with 2 or 4 people?
- How can we show that parts of a whole are equal in size?
- What name would you give part?

- How many fourths make a whole? How many halves make a whole?
- What happens to the parts of a whole as you break them into more parts?

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Common Core State Standards for Mathematical Content

Geometry

1.G

Reason with shapes and their attributes.

1.G.3 Partition circles and rectangles into two and four equal shares, describe the shares using the words *halves*, *fourths*, and *quarters*, and use the phrases *half of*, *fourth of*, and *quarter of*. Describe the whole as two of, or four of the shares. Understand for these examples that decomposing into more equal shares creates smaller shares.

Common Core Standards for Mathematical Practice

1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

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Clarifying the Standards

Prior Learning

Kindergarteners compose simple shapes to create larger shapes. They learn to recognize that shapes can be used to compose other shapes.

Current Learning

In grade 1, students divide shapes into two and four equal parts. They relate new language as they label these parts as fourths, quarters, and halves. Students understand that parts become smaller as they are partitioned into more parts.

Future Learning

Third grade students understand and use numeric representations to label the parts of a whole. They understand that the size of a fractional part is related to the whole. They compare and order fractions. Students apply their knowledge to points on a number line. Fourth graders develop an understanding the connections between fractions and decimals. By the end of elementary school, students are fluent in addition and subtraction of fractions and decimals.

Additional Findings

In addition to work with whole numbers, young students should also have some experience with simple fractions through connections to everyday situations and meaningful problems, starting with the common fractions expressed in the language they bring to the classroom such as "half". At this level, it is more important for students to recognize when things are divided into equal parts than to focus on fractions notation. (*Principles and Standards for School Mathematics*, p. 82)

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